Cracking an Open Safe: HAZUS Vulnerability Functions in Terms of Structure-Independent Spectral Acceleration

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The HAZUS-MH Technical Manual provides a method and data for calculating earthquake loss to ordinary buildings using in part the capacity spectrum method (CSM) of structural analysis, but it does not provide tabular results relating loss to structure-independent intensity measures such as $S_a(0.3\ \text{sec}, 5\%)$ or $S_a(1.0\ \text{sec}, 5\%)$, and no procedure for doing so is offered. It is a minor challenge to perform HAZUS-MH loss calculations outside of HAZUS-MH, owing to the sometimes iterative nature of CSM. A technique to calculate mean loss (here, casualty rates in ordinary buildings) as a function of site-soil-adjusted $S_a(0.3\ \text{sec}, 5\%)$ and $S_a(1.0\ \text{sec}, 5\%)$ is presented that honors all HAZUS-MH methods and data. The resulting seismic vulnerability functions are available at www.risk-agora.org as a resource for open risk modeling. Such vulnerability functions facilitate loss analyses by de-coupling the calculation of hazard from that of loss given hazard. [DOI: 10.1193/1.3106680]

INTRODUCTION

PAGER

The U.S. Geological Survey is adding post-earthquake fatality estimation to its Prompt Assessment of Global Earthquakes for Response (PAGER) program. PAGER's goal is to inform early and rapid post-earthquake decisions about humanitarian aid before ground-truth and news information can be acquired. It can also be used to examine hypothetical scenarios for risk-management purposes. In its post-earthquake mode, PAGER monitors the USGS's near-real-time global earthquake solutions, automatically identifies possibly important events, and estimates the population exposed to various levels of shaking intensity. To enhance those capabilities, the PAGER team is developing several candidate loss models capable of estimating shake-related deaths and other impacts.

One candidate is an analytical model that employs HAZUS-MH motion-damage relationships together with an estimated global building stock and the PAGER-estimated population exposed to various levels of shaking intensity in any given earthquake, anywhere in the world. The HAZUS-MH motion-damage methodology and an extensive associated data set are clearly documented by Kircher et al. (1997) and NIBS and FEMA

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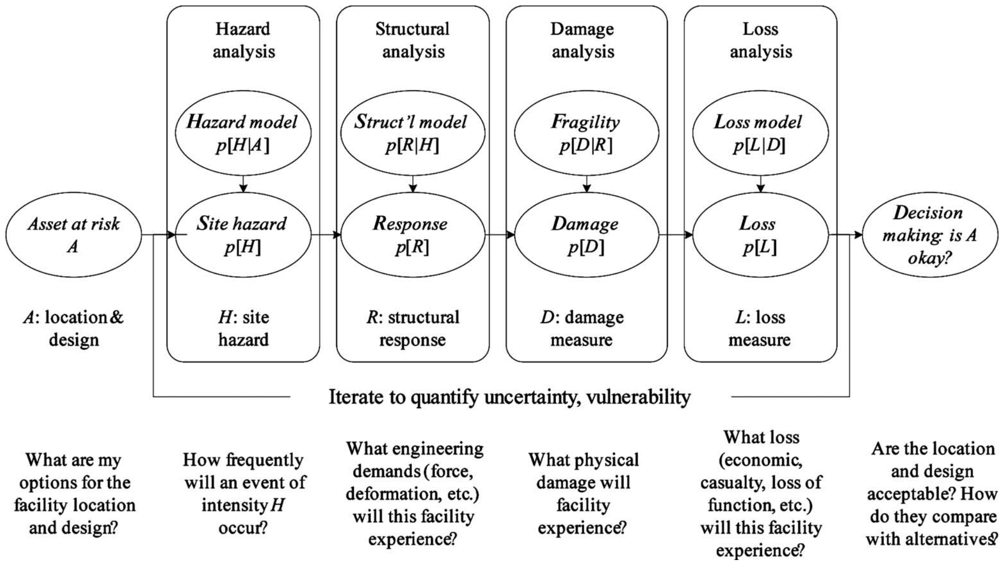

Figure 1. General overview of PBEE-2 methodologies.

(2003), the "open safe" of the title. However, loss is not expressed as functions of a constant-period, constant-damping spectral acceleration response parameter, which tends to require a computationally costly, iterative calculation of structural response, damage, and loss for each building type at each location in each earthquake. The problem addressed here is to crack that open safe, i.e., to convert the HAZUS-MH motion-damage information to tabular vulnerability functions that directly give mean fraction of occupants killed as a function of 5%-damped spectral acceleration response at either 0.3 sec or 1.0 sec period. Of course, such tabular vulnerability functions are useful for other loss-estimation purposes besides PAGER.

OVERVIEW OF THE HAZUS-MH METHODOLOGY FOR ESTIMATING LOSS

Like some other second-generation performance-based earthquake engineering methods that attempt to estimate performance in terms of dollars, deaths, and downtime, HAZUS-MH estimates losses in four analytical stages, illustrated in Figure 1: hazard analysis, structural analysis, damage analysis, and loss analysis. In the hazard analysis, one characterizes the seismic effects at a given site in a parametric form using peak ground acceleration (PGA), peak ground velocity (PGV), and the 5%-damped elastic spectral acceleration response at 0.3 sec and 1.0 sec period on NEHRP site class B (denoted here by S_S and S_1, respectively). These are used to approximate a complete response spectrum as shown in Figure 2a, referred to as the *input spectrum*. For ordinary buildings, only S_S and S_1 matter, so henceforth PGA and PGV are mostly ignored.

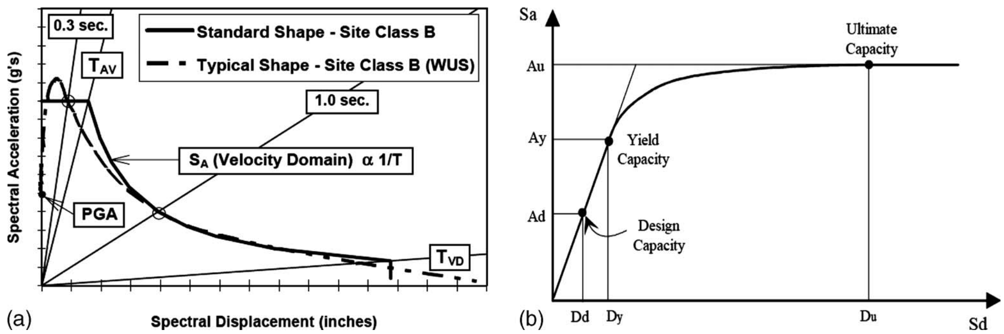

Figure 2. HAZUS-MH standardized response spectrum shape and capacity curve (NIBS and FEMA 2003 Figures 4b and 5d).

In the structural analysis, HAZUS-MH treats a building as a single-degree-of-freedom nonlinear damped oscillator with the force-deformation behavior shown in Figure 2b: a pushover curve (referred to as a *capacity curve*) with a linear portion, an elliptical degrading-stiffness portion, and a perfectly plastic portion. The curve is therefore fully defined by its yield and ultimate capacity points, tabulated in NIBS and FEMA (2003) for each of the 128 combinations of model building type, rise type (low-, mid-, and highrise), and code eras (pre, low, moderate, and high code) by which HAZUS-MH characterizes ordinary buildings. Occupancy also matters to repair cost and other forms of loss, but since the concern here is fatality rate, occupancy is ignored hereafter.

Note that the pushover curve is expressed in the same space as the index spectrum, with spectral acceleration (S_a) in place of force and spectral displacement (S_d) in place of deformation. Beyond yield, hysteretic energy dissipation adds to effective damping. The point where the capacity curve meets a response spectrum (adjusted to account for site soil amplification and hysteretic energy dissipation) is referred to as the *performance point*, which gives the structural response of the building to the earthquake in question.

In the damage analysis, the S_d or S_a of the performance point is input to a set of fragility functions that give the probability that each of three building components is in each of several damage states. The three components are the structural system (assumed to be sensitive to S_d), the portion of nonstructural components sensitive to S_d, and the portion of nonstructural components sensitive to S_a. In the loss analysis, the loss conditioned on each component's damage state is integrated with the probability of that component being in that damage state, which was calculated in the damage analysis. The loss that is of interest here is fatality rate: mean fraction of indoor occupants killed.

The capacity spectrum method of structural analysis has been criticized by several authors who question its accuracy. Nonlinear dynamic analysis has been employed in loss estimation by researchers of the Pacific Earthquake Engineering Research Center, the Applied Technology Council, and others (e.g., Porter et al. 2001, Vamvastikos and Cornell 2002, etc.). But until those methods are integrated with the necessarily extensive structural, damage and loss analysis data, HAZUS-MH remains the only open loss-estimation technique that can be immediately and inexpensively applied to virtually all US construction.

OBJECTIVE

Given S_S and S_1, NIBS and FEMA (2003) provide all the relevant parameter values required to calculate mean loss, but do not combine those parameters to provide simple tabular relationships between mean fatality rate and a structure-independent intensity measure such as S_S, S_1, or a vector combination. This paper shows how such a tabular vulnerability function can be created without iterating to determine the performance point from the input spectrum and the pushover curve. The key is to work backward and calculate the parameters of the input spectrum from the performance point, and then to work forward from the performance point to the expected value of loss, thereby relating the input spectrum to loss. Details of this calculation are provided below.

Let us begin by considering how this backward calculation can be performed. After that the calculation of mean loss given S_d (because only S_d matters to casualties in HAZUS) is presented, and related back to S_S and S_1. Finally, a URL is offered for tabular results for each of the 128 combinations of HAZUS-MH model building type, rise type, and code era, 4 magnitudes, 5 NEHRP site classes, 2 seismic domains (western U.S., denoted WUS, and central and eastern U.S., denoted CEUS), and 4 distances, for a total of 25,600 seismic vulnerability functions giving mean indoor fatality rate as a function of structure-independent, 5%-damped S_a.

The large number of vulnerability functions is an artifact of the HAZUS-MH methodology, not any decision of the author. To make practical use of them requires only one of several free or commercial database applications and a simple query: select the records corresponding to the desired model building type, rise type, magnitude, site class, etc., and sort them in order of increasing S_a.

HAZARD AND STRUCTURAL ANALYSES

The input spectrum. The input spectrum is an idealized response spectrum on site class B, 5% damping. Illustrated in Figure 3, it is given by

$$
\begin{aligned}
S_a &= PGA & T &= 0 \\
&= S_S & 0 &< T \leqslant T_{AV0} \\
&= S_1/T & T_{AV0} &\leqslant T < T_{VD0} \\
&= (S_1 T_{VD0})/T^2 & T &\geqslant T_{VD0}
\end{aligned}
\tag{1}
$$

where PGA is the peak ground acceleration on NEHRP site class B, T is period, T_{AV0} denotes the period at the intersection of the constant-acceleration and constant-velocity portions of the input spectrum, and T_{VD0} denotes the period at the intersection of the constant-velocity and constant-displacement portions of the input spectrum. Equation 1b represents the constant-acceleration portion of the input spectrum; Equation 1c, the constant-velocity portion; and Equation 1d, the constant-displacement portion, the last

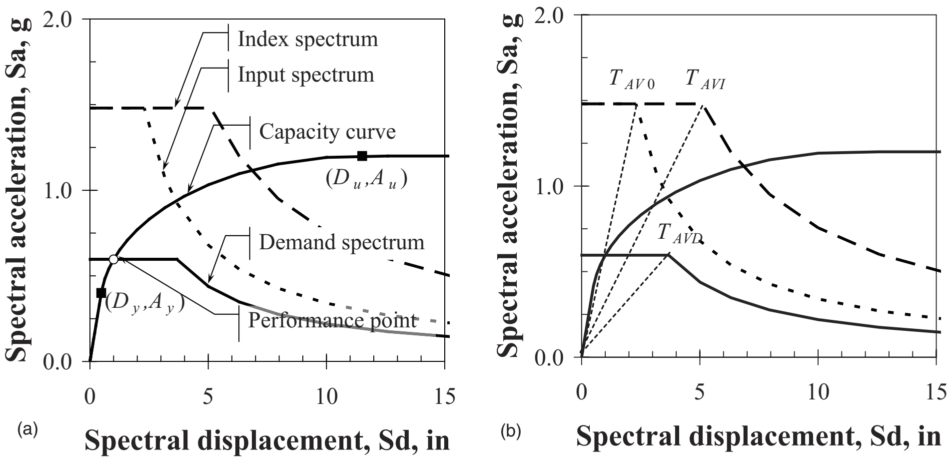

Figure 3. HAZUS-MH capacity curve and various response spectra used here (a), and corner periods used here (b).

of which "does not usually affect calculation of building damage" (Kircher et al. 1997) and is therefore ignored here. One can eliminate the period term by noting that

$$T = 0.32\sqrt{S_d/S_a} \tag{2}$$

where S_d and S_a are measured in inches and gravities, respectively. Substituting 2 into (1)c and ignoring the constant-displacement portion of the spectrum, the result is

$$\begin{aligned} S_a &= PGA && S_d = 0 \\ &= S_S && 0 < S_d \leqslant S_{dAV0} \\ &= S_1^2/(0.102 S_d) && S_{dAV0} \leqslant S_d \end{aligned} \tag{3}$$

where S_{dAV0} indicates the spectral displacement at the intersection of the constant-acceleration and constant-velocity portions of the input spectrum. One can calculate it by equating Equations 3b and 3c, resulting in

$$S_{dAV0} = S_1^2/(0.102 S_S) \tag{4}$$

One can calculate T_{AV0} by equating Equation 1b and 1c:

$$T_{AV0} = S_1/S_S \tag{5}$$

The HAZUS-MH authors provide a table of the inverse of this ratio, called the spectral acceleration response factor (SARF). See Table 1 for the western US and Table 2 for the central and eastern US. The inverse of the SARF suggests that T_{AV0} is roughly 0.3 sec for M=6, 0.4 sec for M=7, and 0.6 sec for M=8 for the western US. For the central and eastern US, T_{AV0} is approximately 0.3 sec for M=6 and 7, 0.4 sec for M=8.

Table 1. Spectral acceleration response factors, WUS rock (Site Class B)

Closest distance to fault rupture	S_S/PGA_B given Magnitude, M:				S_S/S_1 given Magnitude, M:			
	≤5	6	7	≥8	≤5	6	7	≥8
≤10 km	1.4	1.8	2.1	2.1	5.3	3.7	3.1	1.8
20 km	1.5	2.0	2.1	2.0	5.0	3.5	2.5	1.7
40 km	1.6	2.1	2.2	2.0	4.6	3.3	2.3	1.6
≥80 km	1.3	1.8	2.1	2.0	4.1	3.1	2.1	1.5

The demand spectrum. Now consider the demand spectrum, which is of the same form as Equation 1, but factored to account for site class and damping ratio other than 5%. Ignoring the constant-displacement portion of the spectrum, the demand spectrum is given by

$$\begin{aligned} S_a &= PGA_X = PGA \cdot F_a && T = 0 \\ &= S_S F_a / R_A && 0 < T \leqslant T_{AVD} \\ &= S_1 F_v / (R_V T) && T_{AVD} \leqslant T \end{aligned} \tag{6}$$

where PGA_X denotes site-soil-amplified peak ground acceleration and T_{AVD} indicates the period at the intersection of the constant-acceleration and constant-velocity portions of the demand spectrum. The terms F_a and F_v reflect site soil amplification as in ASCE-7 Tables 11.4-1 and 11.4-2 (ASCE 2006) while R_A and R_V account for damping ratio other than 5%:

$$R_A = 2.12/(3.21 - 0.68 \ln[100B_{eff}]) \tag{7}$$

$$R_V = 1.65/(2.31 - 0.41 \ln[100B_{eff}]) \tag{8}$$

where B_{eff} denotes the effective damping ratio (expressed as a fraction, not percent). Substituting 2 into (6)c, the demand spectrum becomes:

Table 2. Spectral acceleration response factors, CEUS rock (Site Class B)

Hypocentral Distance	S_S/PGA_B given Magnitude, M:				S_S/S_1 given Magnitude, M:			
	≤5	6	7	≥8	≤5	6	7	≥8
≤10 km	0.9	1.2	1.5	2.1	8.7	4.2	3.1	2.3
20 km	1.0	1.3	1.4	1.6	8.1	4.0	3.0	2.7
40 km	1.2	1.4	1.6	1.6	7.3	3.7	2.8	2.6
≥80 km	1.5	1.7	1.8	1.9	6.5	3.3	2.5	2.4

$$\begin{aligned} S_a &= PGA \cdot F_a = PGA_X && T = 0 \\ &= S_S F_a / R_A && 0 < T \leqslant T_{AVD} \\ &= S_1^2 F_v^2 / (0.102 R_V^2 S_d) && T_{AVD} \leqslant T \end{aligned} \tag{9}$$

To calculate T_{AVD}, equate Equations (9)b and (9)c, and solve for S_d:

$$S_S F_a / R_A = S_1^2 F_v^2 / (0.102 R_V^2 S_{dAVD})$$

$$S_{dAVD} = \frac{S_1^2 F_v^2 R_A}{0.102 R_V^2 S_S F_a} \tag{10}$$

where S_{dAVD} is the value of S_d at the boundary. Denoting by S_{aAVD} the corresponding spectral acceleration, the period at the boundary is given by

$$T_{AVD} = 0.32 \sqrt{S_{dAVD} / S_{aAVD}} \tag{11}$$

Substituting Equations 9 and 10b into Equation 11 yields

$$T_{AVD} = 0.32 \sqrt{\frac{S_1^2 F_v^2 R_A / (0.102 R_V^2 S_S F_a)}{S_S F_a / R_A}} = 0.32 \sqrt{\frac{S_1^2 F_v^2 R_A^2}{0.102 R_V^2 S_S^2 F_a^2}} = \frac{S_1 F_v R_A}{S_S F_a R_V} \tag{12}$$

Since the HAZUS-MH developers provide this ratio only for NEHRP site class B and 5% damping ratio (see Table 1), the author calculated T_{AVD} explicitly for every combination of magnitude (5, 6, 7, and 8), distance (10 km, 20 km, 40 km, and 80 km), NEHRP site soil classification (A, B,…E), and effective damping ratio (0.05, 0.06,…1.00). For WUS sites, $S_1 F_v$ and $S_S F_a$ were calculated using the Boore et al. (1997) attenuation relationship (mechanism unspecified) with its built-in treatment of Vs30. For CEUS, Toro et al. (1997) was used, with R_M modified per the HAZUS-MH methodology and with the ASCE-7 site-amplification factors F_a and F_v. Shearwave velocities for A, B,…E used in Boore et al. (1997) are taken as 2100, 1070, 520, 250, and 125 m/sec, respectively. shows sample results. To calculate T_{AVD} one could use next-generation attenuation (NGA) relationships, but (1) NGA relationships for CEUS are still in development, (2) consistency with HAZUS-MH is an important objective here, and (3) the point is simply to calculate the period T_{AVD}, not shaking intensities for their own sake. The potential effect of using NGA on T_{AVD} is unknown.

Examination of Table 3 shows that magnitude has a significant impact on T_{AVD}: higher magnitude is associated with higher T_{AVD}, with M=8 potentially having triple the T_{AVD} as M=5 regardless of damping ratio, site class, and seismic domain. WUS events tend to have larger T_{AVD} than CEUS events with similar parameter values. Distance tends to have modest effect of T_{AVD}, and as might be expected soil tends to have larger T_{AVD} than rock. So far, we have simply derived the form of the input and demand spectra, including the period at which the constant-acceleration and constant-velocity portions intersect, for various combinations of magnitude, distance, seismic domain, site class, and damping ratio. It will be useful later to express a third "index" spectrum: the 5%-damped, site-soil-adjusted response spectrum, which is the same as the demand spectrum at 5% damping.

Table 3. Sample T_{AVD} on (a) site class B, western US; (b) site class D, western US, (c) site class B central and eastern US, and (d) site class D, central and eastern US, for 10% damping. Other values apply for different effective damping.

(a)	Magnitude				(b)	Magnitude			
Dist (km)	5	6	7	8	Dist (km)	5	6	7	8
10	0.20	0.23	0.34	0.65	10	0.46	0.54	0.72	1.5
20	0.20	0.23	0.34	0.65	20	0.45	0.54	0.71	1.2
40	0.21	0.24	0.35	0.68	40	0.48	0.55	0.76	1.1
80	0.22	0.25	0.37	0.72	80	0.50	0.58	0.86	1.3
(c)	Magnitude				(d)	Magnitude			
Dist (km)	5	6	7	8	Dist (km)	5	6	7	8
10	0.14	0.27	0.38	0.40	10	0.22	0.44	0.59	0.59
20	0.15	0.28	0.39	0.41	20	0.22	0.43	0.59	0.62
40	0.16	0.30	0.41	0.43	40	0.24	0.44	0.61	0.64
80	0.17	0.32	0.45	0.47	80	0.26	0.49	0.68	0.68

The index spectrum. In this case, in Equation 9, $R_A = R_V = 1$. Again ignoring the constant-displacement portion of the spectrum,

$$\begin{aligned} S_a &= PGA \cdot F_a && T = 0 \\ &= S_S F_a && 0 < T \leq T_{AVI} \\ &= S_1^2 F_v^2 / (0.102 S_d) && T_{AVI} \leq T \end{aligned} \tag{13}$$

where T_{AVI} is the period at which the constant-acceleration and constant-velocity portions of the index spectrum intersect; it is simply T_{AVD} calculated by Equation 12 for 5% damping.

Inferring index spectral parameters from a performance point. The derivation so far gives the various response spectra, including the period at which the constant-acceleration and constant-velocity portions intersect. Now we get to the important trick to avoiding iteration in the structural analysis: calculating backwards from the performance point to the parameters of the input spectrum. Let us define the performance point as the intersection of the capacity curve and the demand spectrum with known values of (S_d, S_a, B_{eff}, X), where X denotes site soil class. For the moment, simply remember that the performance point is on the demand spectrum. Let T denote the period at the performance point.

It is desirable to infer the "control points" of the index spectrum given a point on the demand spectrum (the performance point), magnitude, distance, and site class. Control points here mean $PGA \cdot F_a$ (the peak ground acceleration on the index spectrum), $S_S F_a$

Table 4. Inferring F_a from $S_S F_a$ and site class

Site class	$S_S F_a$, g						
	0.1	0.2	0.4	0.6	0.8	1	1.25
A	0.80	0.80	0.80	0.80	0.80	0.80	0.80
B	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C	1.20	1.20	1.20	1.20	1.11	1.00	1.00
D	1.60	1.60	1.60	1.47	1.30	1.15	1.00
E	2.50	2.50	2.50	2.50	1.88	0.90	0.90

(the spectral acceleration on the constant-acceleration portion of the index spectrum) and S_1F_v (spectral velocity on the constant-velocity portion of the index spectrum). Remember that unless the B_{eff}=5%, the performance point is *not* on the index spectrum. Two cases are of interest:

Case 1: $T \leq T_{AVD}$

Case 2: $T_{AVD} < T$

To determine which case applies, one can calculate T using Equation 2 and then compare with T_{AVD} as discussed above. In case 1, begin by calculating the 5%-damped spectral acceleration on the constant-acceleration portion of the index spectrum, as follows. From Equation 9, and then substituting for R_A from Equation 7,

$$S_S F_a = S_a R_A = 2.12 S_a / (3.21 - 0.68 \ln[100 B_{eff}]) \quad \text{if } T \leqslant T_{AVD} \tag{14}$$

To infer S_1 from the results of Equation 14, one multiplies $S_S F_a$ by two factors:

$$S_1 = S_S F_a \cdot \frac{1}{(S_S/S_1)} \cdot \frac{1}{F_a(S_S F_a)} \tag{15}$$

Then using S_1 calculated from Equation 15, one arrives at the site-amplified, 5%-damped 1-second spectral acceleration response associated with the performance point:

$$S_1 F_v = S_S F_a \cdot \frac{1}{(S_S/S_1)} \cdot \frac{1}{F_a(S_S F_a)} \cdot F_v(S_1) \tag{16}$$

In Equations 15 and 16, S_S/S_1 is the spectral acceleration response factor (SARF) discussed above. One could use either the HAZUS-supplied values of SARF or those derived here for T_{AVD} for NEHRP site class B and 5% damping. For consistency with HAZUS-MH, the HAZUS-MH values are used here for S_S/S_1. The third term on the right-hand side of Equation 15 is the site amplification factor F_a expressed as a function of $S_S F_a$, as opposed to S_S. To express F_a as a function of $S_S F_a$ may seem counterintuitive, but it is simply a mapping from site-amplified shaking to rock shaking, rather than the reverse. Table 4 contains this mapping. It is calculated by multiplying S_S by F_a for each S_S level in ASCE (2006) Table 11.4-1 (the ASCE-7 table of F_a), then linearly in-

Table 5. Inferring F_v from S_1F_v and site class

Site class	S_1F_v, g						
	0.1	0.2	0.4	0.6	0.8	1	1.2
A	0.80	0.80	0.80	0.80	0.80	0.80	0.80
B	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C	1.70	1.68	1.54	1.36	1.30	1.30	1.30
D	2.40	2.40	2.00	1.68	1.50	1.50	1.50
E	3.50	3.50	3.45	3.24	2.88	2.40	2.40

terpolating (S_SF_a, F_a) at various fixed values of S_SF_a. The last term on the right-hand side of Equation 16 is the site amplification factor F_v as a function of S_1, from the ASCE-7 table for F_v, i.e., Table 11.4-2.

In case 2, one calculates the 5%-damped spectral acceleration at 1-second period on the index spectrum. From Equation 9, and then substituting for R_V from Equation 8,

$$S_1F_v = 0.32R_V\sqrt{S_aS_d} = 0.528 \cdot \sqrt{S_aS_d}/(2.31 - 0.41\ \ln[100B_{\text{eff}}]) \quad \text{if } T > T_{AVD} \quad (17)$$

To infer S_SF_a from the results of Equation 17, one follows a similar process to case 1:

$$S_S = S_1F_v \cdot (S_S/S_1) \cdot \frac{1}{F_v(S_1F_v)} \quad (18)$$

$$S_SF_a = S_1F_v \cdot (S_S/S_1) \cdot \frac{1}{F_v(S_1F_v)} \cdot F_a(S_S) \quad (19)$$

Here one multiplies rather than divides by S_S/S_1, divides by F_v (expressed as a function of S_1F_v rather than of S_1; see Table 5) and multiplies by F_a expressed as a function of S_S.

We have now calculated the site-soil-amplified values of $S_a(0.3\text{ sec}, 5\%)$ and $S_a(1.0\text{sec}, 5\%)$ associated with any point on the capacity curve. If desired, site-amplified values of PGA can be calculated as well, by dividing $S_a(0.3\text{ sec}, 5\%)$ by HAZUS' tabulated ratios of S_S to PGA shown in Table 1 and Table 2.

STRUCTURAL ANALYSIS

We have seen how one can infer S_SF_a and S_1F_v for any performance point (S_d, S_A, B_{eff}, X): Calculate T from Equation 2, and if $T \leq T_{AVD}$, calculate S_SF_a and then S_1F_v from Equations 14 and 16, respectively. If $T > T_{AVD}$, calculate S_1F_v and then S_SF_a from Equations 17 and 19, respectively. Let us turn to the calculation of the performance point. As noted earlier, the structural response of ordinary buildings in HAZUS-MH is

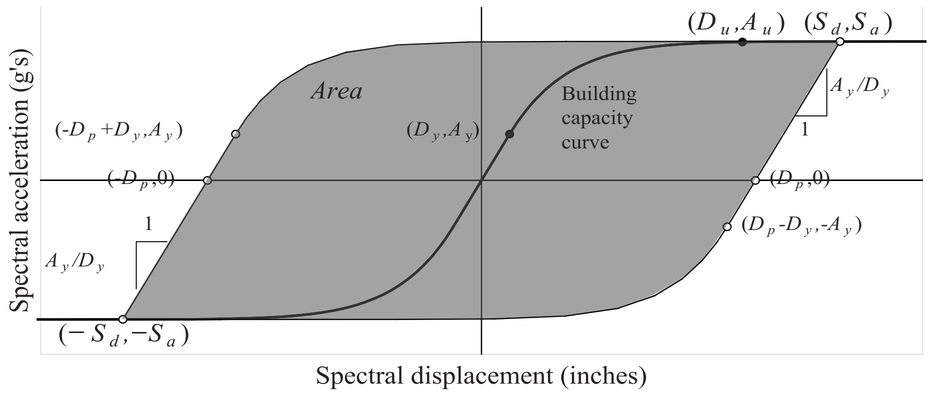

Figure 4. Establishing effective damping ratio at HAZUS-MH performance point.

described by a three-part pushover curve in the space of (S_d, S_a) illustrated in Figure 2: linear from (0,0) to (D_y, A_y), perfectly plastic after (D_u, A_u), and an elliptical spline in between (Bouabid 2007, Lee 2008).

$$\begin{aligned} S_a &= S_d A_y / D_y && S_d < D_y \\ &= A_0 + b\sqrt{1 - \frac{(S_d - D_u)^2}{a^2}} && D_y \leqslant S_d < D_u \\ &= A_u && D_u \leqslant S_d \end{aligned} \tag{20}$$

where

$$b = \frac{D_y(A_y - A_u)^2 - (D_y - D_u)A_y(A_y - A_u)}{(D_y - D_u)A_y - 2D_y(A_y - A_u)}$$

$$a = \sqrt{\frac{-D_y(D_y - D_u)b^2}{A_y(A_y - A_u + b)}}$$

$$A_0 = A_u - b \tag{21}$$

The effective damping ratio at any point on the capacity curve, denoted by B_{eff}, is given by

$$B_{eff} = B_E + \kappa\left(\frac{Area}{2\pi S_d S_a}\right) \tag{22}$$

where B_E is the elastic damping of the model building type, κ is a degradation factor, and *Area* is the area enclosed by the hysteresis loop as in Figure 4. Ignoring the rounded corners,

$$Area \approx 4S_a\left(S_d - \frac{S_a}{A_y/D_y}\right) \tag{23}$$

Substituting 23 in 22 yields

$$B_{eff} = B_E + \kappa\left(\frac{2}{\pi}\left[1 - \frac{K_s}{K_E}\right]\right) \tag{24}$$

where K_s is the secant stiffness S_a/S_d and K_E is the elastic stiffness A_y/D_y. The capacity curve and the effective damping at any point along it are fully defined by the parameters D_y, A_y, D_u, A_u, B_E, and κ, which are all tabulated in NIBS and FEMA (2003). In fact, one can calculate D_y, A_y, D_u, and A_u from more-basic parameters of design strength, overstrength, ductility, etc., but this is not discussed here. The degradation factor κ also depends on shaking duration, categorized as short, moderate, or long, and assumed to depend in turn on M, with short duration corresponding to $M \leq 5.5$, long duration corresponding to $M \geq 7.5$, and moderate duration corresponding to anything in between. In loss analysis using probabilistic ground motions, no single magnitude pertains, so the HAZUS-MH developers recommend moderate duration shaking for selecting κ.

DAMAGE ANALYSIS

Now we work forward from the performance point to damage and loss. HAZUS-MH treats earthquake casualties solely as a function of structural damage, so nonstructural damage will be ignored henceforth. The structural damage to the building is treated in terms of a single damage state that can take on one of six values: undamaged (denoted here by $d=0$), slight, moderate, or extensive damage (denoted here by $d=1$, 2, and 3, respectively), complete but not collapsed ($d=4$), and collapsed ($d=5$). The probability of each structural damage state is given as a function solely of S_d:

$$\begin{aligned} P[D=d|S_d=x] &= 1-\Phi\left(\frac{\ln[x/\theta_1]}{\beta_1}\right) && d=0 \\ &= \Phi\left(\frac{\ln[x/\theta_d]}{\beta_d}\right) - \Phi\left(\frac{\ln[x/\theta_{d+1}]}{\beta_{d+1}}\right) && 1 \leqslant d \leqslant 3 \\ &= (1-P_c)\Phi\left(\frac{\ln[x/\theta_4]}{\beta_4}\right) && d=4 \\ &= P_c\Phi\left(\frac{\ln[x/\theta_4]}{\beta_4}\right) && d=5 \end{aligned} \tag{25}$$

where $P[D=d|S_d=x]$ denotes the probability of structural damage state d given that S_d takes on some particular value x, and Φ denotes the cumulative standard normal distribution. The parameters θ_i, β_i, and P_c denote, respectively, the median and logarithmic standard deviation values of the component capacity to resist damage state i, and the fraction of buildings in the complete damage state that are expected to be collapsed. They are tabulated in NIBS and FEMA (2003). Note that calculations similar to Equation 25 can be performed to determine the probabilistic damage state of nonstructural components: drift-sensitive components use S_d at the performance point as input, while acceleration-sensitive components use S_a as input, and neither have a damage state 5.

LOSS ANALYSIS

Now let us address loss. The HAZUS-MH developers tabulate mean fatality rate for each structure type and damage state d. Let L_d denote the fatality rate in a building that experiences structural damage state d, and let L denote the total mean fatality rate. One can employ the theorem of total probability and Equation 25 to estimate L as a function of S_d:

$$L = \sum_{d=0}^{5} P[D = d | S_d = x] L_d \tag{26}$$

The same procedure is used to calculate mean rate of nonfatal injuries (there are three nonfatal injury levels considered in HAZUS-MH), employing other L_d values tabulated in NIBS and FEMA (2003). Also, given the damage state to both structural and the nonstructural components, one can calculate mean repair cost using tables of repair cost versus component damage state by occupancy type. Mean repair costs are treated in a companion work.

RELATING LOSS TO SPECTRAL ACCELERATION

We can now consider an algorithm to calculate the mean fatality rate as a function of 5%-damped $S_S F_a$ and $S_1 F_v$.

1. Select a structure type, seismic domain (plate boundary or shield), site class, magnitude range ($M \leq 5.5$, $5.5 < M < 7.5$, or $M \geq 7.5$), approximate distance (<10 km, 20 km, 40 km, >80 km), and a value of S_d. For thoroughness and simplicity the author considered 51 values of S_d equally logarithmically spaced from 0.01 in to 1000 in.
2. Calculate S_a using Equations 20 and 21.
3. Calculate B_{eff} using Equation 24.
4. Calculate T using Equation 2.
5. If $T \leq T_{AVD}$, calculate $S_S F_a$ and $S_1 F_v$ using Equations 14 and 16, respectively. Otherwise calculate $S_1 F_v$ and $S_S F_a$ using Equations 17 and 19, respectively.
6. Calculate damage probabilities using Equation 25.
7. Calculate the mean fatality rate L using Equation 26.
8. Repeat steps 2 through 7 for each value of S_d, calculating $S_1 F_v$, $S_S F_a$, and L. If desired, interpolate $(S_1 F_v, L)$ and $(S_S F_a, L)$ at a common set of shaking intensities such as $0.10, 0.20, \ldots 4.0$ g.
9. Repeat steps 1 through 8 for all combinations of seismic domain, site class, magnitude range, and structure type of interest.

Table 6. Capacity-curve parameters for high-code W1

D_y	A_y	D_u	A_u	B_E	κ_{short}	κ_{med}	κ_{long}
0.48	0.40	11.51	1.2	0.175	1.0	0.8	0.5

SAMPLE CALCULATIONS AND RESULTS

These calculations were performed in a Microsoft Access database. The results are too voluminous to provide in their entirety here, but a sample can be presented, and full results tabulated at www.risk-agora.org. Consider a high-code woodframe, single-family dwelling (type W1) on a western US site, with NEHRP site class D, subjected to $6.5 \leq \mathrm{M} < 7.5$ at distance $15\ \mathrm{km} \leq \mathrm{R} < 30$ km. Consider $S_d = 1.0$ in. Remember that the calculation will produce a fatality rate for each value of a range of S_d, so one can select any value of S_d for illustration purposes. That is, we do not concern ourselves here with estimating the value of S_d that would result from such an earthquake at such a distance; M and R are only used to select the proper values of κ and T_{AVD}. To begin, the capacity-curve parameters for high-code W1 are shown in Table 6, from NIBS and FEMA (2003) Tables 5.7a (the D and A values) and 5.18 (κ values). Elastic damping B_E is taken from Newmark and Hall (1982).

In the following, variables and results are shown only to two or three significant figures, but the calculations are performed without rounding, so calculations with the rounded values might not exactly match the results shown. Since $D_y \leq S_d < D_u$,

$$b = \frac{0.48(0.4-1.2)^2 - (0.48-11.51)0.4(0.4-1.2)}{(0.48-11.51)0.4 - 2 \cdot 0.48(0.4-1.2)} = 0.8843$$

$$a^2 = \frac{-0.48(0.48-11.51)0.8843^2}{0.4(0.4-1.2+0.8843)} = 122.8 \quad \text{from (21)}$$

$$A_0 = 1.2 - 0.8843 = 0.3157$$

$$S_a = 0.3157 + 0.8843\sqrt{1 - \frac{(1.0-11.51)^2}{122.8}} = 0.5958 \quad \text{from (20)}$$

$$Area = 4 \cdot 0.60 \cdot \left(1 - \frac{0.60}{0.4/0.48}\right) = 0.67 \text{ from (23)}$$

$$B_{eff} = 0.175 + 0.8 \cdot \left(\frac{0.67}{2\pi \cdot 1.0 \cdot 0.60}\right) = 0.32 \text{ from (22)}$$

$$T = 0.32\sqrt{1.0/0.60} = 0.41 \quad \text{from (2)}$$

For 32% damping, $M=7$, $R=20$, western US, and NEHRP site class D, $T_{AVD}=0.88$ sec, so $T < T_{AVD}$, and we apply Equations 14–16:

Table 7. Fragility parameters for high-code W1

θ_1	β_1	θ_2	β_2	θ_3	β_3	θ_4	β_4	P_c
0.5	0.8	1.51	0.81	5.04	0.85	12.6	0.97	0.03

$$S_S F_a = 2.12 \cdot 0.5958g/(3.21 - 0.68\ \ln[100 \cdot 0.32]) = 1.48g \quad \text{from (14)}$$

$$S_1 = 1.48 \cdot \frac{1}{2.5} \cdot \frac{1}{1.0} = 0.59 \quad \text{from (15)}$$

$$S_1 F_v = 0.59g \cdot 1.5 = 0.88g \quad \text{from (16)}$$

Now the damage analysis. The fragility parameters for high-code W1 are shown in Table 7, taken from NIBS and FEMA (2003) Table 5.9a. Showing the calculation only for $d=4$,

$$P[D=4|S_d=1.0] = (1-0.03) \cdot \Phi\left(\frac{\ln[1.0/12.6]}{0.97}\right) = 0.00439 \quad \text{from (25)}$$

Denoting the damage-state probability for state i as P_i, the sample results are given in Table 8. Remember the damage state probabilities need not sum to 1.0, because $D=0$ is omitted. Values of L_d are shown in Table 9, from NIBS and FEMA (2003) Tables 13.3 through 13.7. The total fatality rate is calculated as

$$L = \sum_{d=0}^{5} P[D=d|S_d=x]L_d = 0.503 \cdot 0 + 0.276 \cdot 0 + 0.024 \cdot 0.00001 + 0.0044 \cdot 0.0001 + 0.00014 \cdot 0.05 = 7.5 \cdot 10^{-6} \quad \text{from (26)}$$

Repeating the calculations at various values of S_d, one can plot L versus $S_S F_a$ (i.e., the site-soil-adjusted 5%-damped elastic spectral acceleration response at 0.3 sec period), as shown in Figure 5. The dot is the sample calculation point.

CONCLUSIONS

The HAZUS-MH developers clearly document how structural response, damage, and loss are calculated in HAZUS-MH. It is an extensively reviewed and widely accepted procedure, an "open safe." However, the documentation does not provide tabular vulnerability functions in the form of mean loss (fatality rate, damage factor, etc.) as a function

Table 8. Structural damage state probabilities

P_1	P_2	P_3	P_4	P_5
0.503	0.276	0.024	0.0044	0.00014

Table 9. Fatality rates in high-code W1

L_1	L_2	L_3	L_4	L_5
0	0	0.00001	0.0001	0.05

of a structure-independent shaking intensity such as 5%-damped spectral acceleration response at one or more reference periods such as *PGA*, S_a(0.3 sec), S_a(1.0 sec), etc. Developers of the U.S. Geological Survey's Prompt Assessment of Global Earthquake for Response (PAGER) project found it desirable to compile such tabular vulnerability functions, so the HAZUS-MH methodology was "cracked" to infer them. The vulnerability functions should be useful for other applications.

This manuscript documents a technique to create such vulnerability functions, including step-by-step equations and complete reference to the source of all relevant HAZUS-MH parameters. The reader who is modestly adept in the use of a spreadsheet, computer programming, or a database application should be able to duplicate the math or substitute any parameter value or methodological feature. The results using all HAZUS-MH parameters and methods for mean indoor casualty rates versus $S_a(0.3\ \mathrm{sec}, 5\%)$ and $S_a(1.0\ \mathrm{sec}, 5\%)$, are now available for free download from www.risk-agora.org as a resource for open risk modeling under the AGORA framework. Vulnerability functions are tabulated for 128 HAZUS-MH structure types (model building types x code levels), 2 seismic domains (western US and central and eastern US), 5

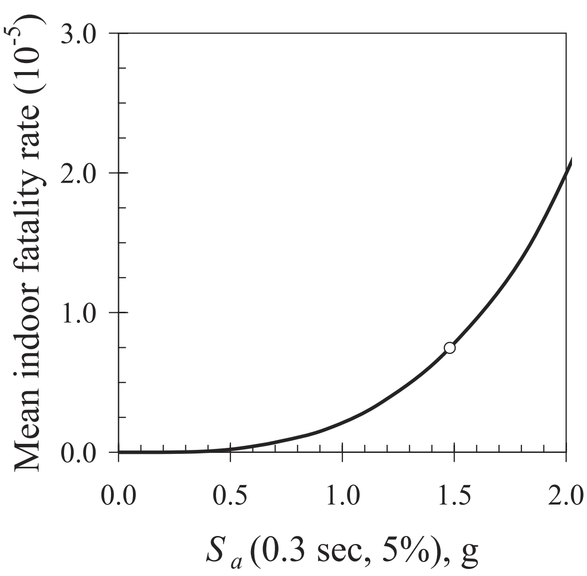

Figure 5. Sample vulnerability function: fatality rate, high-code W1, western US, site class D, moderate-duration events, 15 km$\leq R <$30 km.

Table 10. Sample layout of vulnerability-function table

MBTplus	Domain	M	R	Siteclass	IM	$S_S F_a$	$S_1 F_v$	L1	L2	L3	L4
W1h	WUS	7	20	D	Sa03	1.48	0.88	0.0015	0.00018	4.8E−06	7.5E−06
W1h	WUS	7	20	D	Sa03	1.83	1.1	0.0021	0.00029	9.3E−06	1.5E−06

NEHRP site classes, 16 combinations of magnitude and distance range (relevant to duration, hysteretic energy dissipation, and spectral shape), and 4 injury severity levels. The table is laid out as shown in Table 10.

In the table, "MBTplus" is the HAZUS-MH model building type (e.g., W1, meaning small woodframe) plus a character to indicate code era (e.g., h, meaning high code). "Domain" refers to western US ("WUS") or central and eastern US ("CEUS"). M refers to magnitude and R to distance in km. "Siteclass" refers to the NEHRP site soil classification (A, B, C, D, or E). "IM" indicates whether the performance point corresponds to a point on the constant-acceleration portion of the response spectrum ("Sa03") or on the constant-velocity portion ("Sa10"); if the former, it is best to relate loss to "$S_S F_a$," meaning $S_a(0.3\text{ sec}, 5\%)$, otherwise "$S_1 F_v$" meaning $S_a(1.0\text{ sec}, 5\%)$, both in units of g. Finally, "$L1$" through "$L4$" refer to the mean fraction of building occupants in each of 4 HAZUS-MH injury severity levels ($L4$ being fraction killed). $L1$ through $L4$ are not rounded in the full table, but one should not infer accuracy beyond perhaps one or two significant figures. The table is available as a comma-separated-value text file, with a header indicating its origin and field names in the second line.

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